**Mean free path** is the mean distance traversed by an electron between scattering events. If $\tau$ is the mean free time between scattering events and $u$ is the mean speed of the electron, then the mean free path is $\ell = u\tau$.

**Mean free time** is the average time it takes to scatter a conduction electron. If $t_i$ is the free time between collisions (between scattering events) for an electron labeled $i$, then $\tau = \bar{t}_i$ averaged over all the electrons. The drift mobility is related to the mean free time by $\mu_d = e\tau/m_e$. The reciprocal of the mean free time is the mean probability per unit time that a conduction electron will be scattered; in other words, the mean frequency of scattering events.

**Nordheim's rule** states that the resistivity of a solid solution (an isomorphous alloy) due to impurities $\rho_I$ is proportional to the concentrations of the solute $X$ and the solvent $(1 - X)$.

**Phase** (in materials science) is a physically homogeneous portion of a materials system that has uniform physical and chemical characteristics.

**Relaxation time** is an equivalent term for the mean free time between scattering events.

**Residual resistivity** ($\rho_R$) is the contribution to the resistivity arising from scattering processes other than thermal vibrations of the lattice, for example, impurities, grain boundaries, dislocations, point defects.

**Skin effect** is an electromagnetic phenomenon that, at high frequencies, restricts ac current flow to near the surface of a conductor to reduce the energy stored in the magnetic field.

**Solid solution** is a crystalline material that is a homogeneous mixture of two or more chemical species. The mixing occurs at the atomic scale, as in mixing alcohol and water. Solid solutions can be substitutional (as in Cu–Ni) or interstitial (for example, C in Fe).

**Stefan's law** is a phenomenological description of the energy radiated (as electromagnetic waves) from a surface per second. When a surface is heated to a temperature $T$, it radiates net energy at a rate given by $P_{\text{radiated}} = \epsilon\sigma_S A(T^4 - T_0^4)$, where $\sigma_S$ is Stefan's constant ($5.67 \times 10^{-8}$ W m$^{-2}$ K$^{-4}$), $\epsilon$ is the emissivity of the surface, $A$ is the surface area, and $T_0$ is the ambient temperature.

**Temperature coefficient of resistivity (TCR)** ($\alpha_0$) is defined as the fractional change in the electrical resistivity of a material per unit increase in the temperature with respect to some reference temperature $T_0$.

**Thermal conductivity** ($\kappa$) is a property of a material that quantifies the ease with which heat flows along the material from higher to lower temperature regions. Since heat flow is due to a temperature gradient, $\kappa$ is the rate of heat flow across a unit area per unit temperature gradient.

**Thermal resistance** ($\theta$) is a measure of the difficulty with which heat conduction takes place along a material sample. The thermal resistance is defined as the temperature drop per unit heat flow, $\theta = \Delta T/Q'$. It depends on both the material and its geometry. If the heat losses from the surfaces are negligible, then $\theta = L/\kappa A$, where $L$ is the length of the sample (along heat flow) and $A$ is the cross-sectional area.

**Thermally activated conductivity** means that the conductivity increases in an exponential fashion with temperature as in $\sigma = \sigma_o \exp(-E_\sigma/kT)$ where $E_\sigma$ is the activation energy.

**Thin film** is a conductor whose thickness is typically less than ~1 micron; the thickness is also much less than the width and length of the conductor. Typically thin films have a higher resistivity than the corresponding bulk material due to the grain boundary and surface scattering.

# QUESTIONS AND PROBLEMS

2.1 **Electrical conduction** Na is a monovalent metal (BCC) with a density of 0.9712 g cm$^{-3}$. Its atomic mass is 22.99 g mol$^{-1}$. The drift mobility of electrons in Na is 53 cm$^2$ V$^{-1}$ s$^{-1}$.

*a.* Consider the collection of conduction electrons in the solid. If each Na atom donates one electron to the electron sea, *estimate* the mean separation between the electrons. (Note: If $n$ is the concentration of particles, then the particles' mean separation $d = 1/n^{1/3}$.)

*b.* Estimate the mean separation between an electron ($e^-$) and a metal ion ($Na^+$), assuming that most of the time the electron prefers to be between two neighboring $Na^+$ ions. What is the approximate Coulombic interaction energy (in eV) between an electron and an $Na^+$ ion?

*c.* How does this electron/metal-ion interaction energy compare with the average thermal energy per particle, according to the kinetic molecular theory of matter? Do you expect the kinetic molecular theory to be applicable to the conduction electrons in Na? If the mean electron/metal-ion interaction energy is of the same order of magnitude as the mean *KE* of the electrons, what is the mean speed of electrons in Na? Why should the mean kinetic energy be comparable to the mean electron/metal-ion interaction energy?

*d.* Calculate the electrical conductivity of Na and compare this with the experimental value of $2.1 \times 10^7\ \Omega^{-1}\ m^{-1}$ and comment on the difference.

2.2 **Electrical conduction** The resistivity of aluminum at 25 °C has been measured to be $2.72 \times 10^{-8}\ \Omega$ m. The thermal coefficient of resistivity of aluminum at 0 °C is $4.29 \times 10^{-3}\ K^{-1}$. Aluminum has a valency of 3, a density of 2.70 g $cm^{-3}$, and an atomic mass of 27.

*a.* Calculate the resistivity of aluminum at −40 °C.

*b.* What is the thermal coefficient of resistivity at −40 °C?

*c.* Estimate the mean free time between collisions for the conduction electrons in aluminum at 25 °C, and hence estimate their drift mobility.

*d.* If the mean speed of the conduction electrons is about $2.0 \times 10^6$ m $s^{-1}$, calculate the mean free path and compare this with the interatomic separation in Al (Al is FCC). What should be the thickness of an Al film that is deposited on an IC chip such that its resistivity is the same as that of bulk Al?

*e.* What is the percentage change in the power loss due to Joule heating of the aluminum wire when the temperature drops from 25 °C to −40 °C?

2.3 **Conduction in gold** Gold is in the same group as Cu and Ag. Assuming that each Au atom donates one conduction electron, calculate the drift mobility of the electrons in gold at 22 °C. What is the mean free path of the conduction electrons if their mean speed is $1.4 \times 10^6$ m $s^{-1}$? (Use $\rho_o$ and $\alpha_o$ in Table 2.1.)

2.4 **Mean free time between collisions** Let $1/\tau$ be the mean probability per unit time that a conduction electron in a metal collides with (or is scattered by) lattice vibrations, impurities, or defects, etc. Then the probability that an electron makes a collision in a small time interval $\delta t$ is $\delta t/\tau$. Suppose that $n(t)$ is the concentration of electrons that have not yet collided. The change $\delta n$ in the uncollided electron concentration is then $-n\delta t/\tau$. Thus, $\delta n = -n\delta t/\tau$, or $\delta n/n = -\delta t/\tau$. We can integrate this from $n = n_o$ at $x = 0$ to $n = n(t)$ at time $t$ to find the concentration of uncollided electrons $n(t)$ at $t$

*Concentration of uncollided electrons*

$$n(t) = n_o\exp(-t/\tau) \qquad [2.84]$$

Show that the mean free time and mean square free time are given by

*Electron scattering statistics*

$$\bar{t} = \frac{\int_0^\infty t\,n(t)dt}{\int_0^\infty n(t)dt} = \tau \quad \text{and} \quad \overline{t^2} = \frac{\int_0^\infty t^2 n(t)dt}{\int_0^\infty n(t)dt} = 2\tau^2 \qquad [2.85]$$

What is your conclusion?

2.5 **Effective number of conduction electrons per atom**

*a.* Electron drift mobility in tin (Sn) is 3.9 $cm^2\ V^{-1}\ s^{-1}$. The room temperature (20 °C) resistivity of Sn is about 110 nΩ m. Atomic mass $M_{at}$ and density of Sn are 118.69 g $mol^{-1}$ and 7.30 g $cm^{-3}$, respectively. How many "free" electrons are donated by each Sn atom in the crystal? How does this compare with the position of Sn in Group IVB of the Periodic Table?

*b.* Consider the resistivity of few selected metals from Groups I to IV in the Periodic Table in Table 2.8. Calculate the number of conduction electrons contributed per atom and compare this with the location of the element in the Periodic Table. What is your conclusion?

*2.12 **TCR and alloy resistivity** Table 2.13 shows the resistivity and TCR ($\alpha$) of Cu–Ni alloys. Plot TCR versus $1/\rho$, and obtain the best-fit line. What is your conclusion? Consider the Matthiessen rule, and explain why the plot should be a straight line. What is the relationship between $\rho_{Cu}$, $\alpha_{Cu}$, $\rho_{CuNi}$, and $\alpha_{CuNi}$? Can this be generalized?

**Table 2.13** Cu–Ni alloys, resistivity, and TCR

| | Ni wt.% in Cu–Ni | | | | |
|---|---|---|---|---|---|
| | 0 | 2 | 6 | 11 | 20 |
| Resistivity (nΩ m) | 17 | 50 | 100 | 150 | 300 |
| TCR (ppm °C$^{-1}$) | 4270 | 1350 | 550 | 430 | 160 |

NOTE: ppm-parts per million, *i.e.*, $10^{-6}$.

2.13 **Hall effect measurements** The resistivity and the Hall coefficient of pure aluminum and Al with 1 at.% Si have been measured at 20 °C (293 K) as $\rho$ = 2.65 μΩ cm, $R_H = -3.51 \times 10^{-11}$ m$^3$ C$^{-1}$ for Al and $\rho$ = 3.33 μΩ cm, $R_H = -3.16 \times 10^{-11}$ m$^3$ C$^{-1}$ for 99 at.% Al-1 at% Si. The lattice parameters for the pure metal and the alloy are 0.4049 nm and 0.4074 nm. What does the simple Drude model predict for the drift mobility in these two metals? How many conduction electrons are there per atom? (Data from M Bradley and John Stringer, J. Phys. F: Metal Phys., *4,* 839, 1974).

2.14 **Hall effect and the Drude model** Table 2.14 shows the experimentally measured Hall coefficient and resistivities for various metals and their position in the periodic table. (a) Calculate the Hall mobility of each element. (b) Calculate the conduction electron concentration from the experimental value of $R_H$. (c) Find how many electrons per atom are contributed to the conduction electron gas in the metal per metal atom. What is your conclusion?

**Table 2.14** Measured Hall coefficients for a few metals at 25 °C

| | Li | Na | K | Cs | Cu | Ag | Au | Ca | Mg | Zn | Al | In |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Group | I | I | I | I | IB | IB | IB | IIA | IIA | IIB | III | III |
| $R_H$ (×10$^{-11}$ m$^3$ C$^{-1}$) | −15 | −24.8 | −42.8 | −73.3 | −5.4 | −9.0 | −7.2 | −17.8 | −8.3 | +10.4 | −3.4 | −0.73 |
| $\rho$ (nΩ m) | 92.8 | 48.8 | 73.9 | 208 | 17.1 | 16.7 | 22.6 | 33.6 | 44.8 | 60.1 | 27.1 | 83.7 |

SOURCE: Hurd, C., *The Hall Coefficient of Metals and Alloys,* Plenum, New York, NY, 1972, along with other sources.

2.15 **The Hall effect** Consider a rectangular sample, a metal or an $n$-type semiconductor, with a length $L$, width $W$, and thickness $D$. A current $I$ is passed along $L$, perpendicular to the cross-sectional area $WD$. The face $W \times L$ is exposed to a magnetic field density $B$. A voltmeter is connected across the width, as shown in Figure 2.40, to read the Hall voltage $V_H$.

*a.* Show that the Hall voltage recorded by the voltmeter is

*Hall voltage*

$$V_H = \frac{IB}{Den}$$

*b.* Consider a 1-micron-thick strip of gold layer on an insulating substrate that is a candidate for a Hall probe sensor. If the current through the film is maintained at constant 100 mA, what is the magnetic field that can be recorded per μV of Hall voltage?

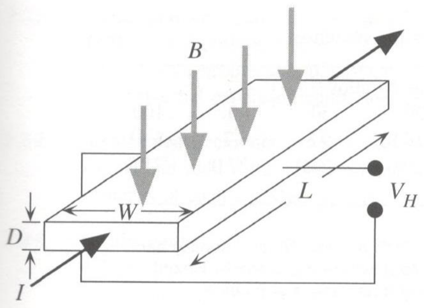


**Figure 2.40** Hall effect in a rectangular material with length $L$, width $W$, and thickness $D$.
The voltmeter is across the width $W$.

**2.16** **Electrical and thermal conductivity of In** Electron drift mobility in indium has been measured to be 6 cm$^2$ V$^{-1}$ s$^{-1}$. The room temperature (27 °C) resistivity of In is $8.37 \times 10^{-8}$ Ωm, and its atomic mass and density are 114.82 amu or g mol$^{-1}$ and 7.31 g cm$^{-3}$, respectively.

*a.* Based on the resistivity value, determine how many free electrons are donated by each In atom in the crystal. How does this compare with the position of In in the Periodic Table (Group IIIB)?

*b.* If the mean speed of conduction electrons in In is $1.74 \times 10^8$ cm s$^{-1}$, what is the mean free path?

*c.* Calculate the thermal conductivity of In. How does this compare with the experimental value of 81.6 W m$^{-1}$ K$^{-1}$?

**2.17** **Electrical and thermal conductivity of Ag** The electron drift mobility in silver has been measured to be 54 cm$^2$ V$^{-1}$ s$^{-1}$ at 27 °C. The atomic mass and density of Ag are given as 107.87 amu or g mol$^{-1}$ and 10.50 g cm$^{-3}$, respectively.

*a.* Assuming that each Ag atom contributes one conduction electron, calculate the resistivity of Ag at 27 °C. Compare this value with the measured value of $1.6 \times 10^{-8}$ Ωm at the same temperature and suggest reasons for the difference.

*b.* Calculate the thermal conductivity of silver at 27 °C and at 0 °C.

**2.18** **Mixture rules** A 70% Cu–30% Zn brass electrical component has been made of powdered metal and contains 15 vol.% porosity. Assume that the pores are dispersed randomly. Given that the resistivity of 70% Cu–30% Zn brass is 62 nΩ m, calculate the effective resistivity of the brass component using the simple conductivity mixture rule, Equation 2.32, and the Reynolds and Hough rule.

**2.19** **Mixture rules**

*a.* A certain carbon electrode used in electrical arcing applications is 47 percent porous. Given that the resistivity of graphite (in polycrystalline form) at room temperature is about 9.1 $\mu$Ω m, estimate the effective resistivity of the carbon electrode using the appropriate Reynolds and Hough rule and the simple conductivity mixture rule. Compare your estimates with the measured value of 18 $\mu$Ω m and comment on the differences.

*b.* Silver particles are dispersed in a graphite paste to increase the effective conductivity of the paste. If the volume fraction of dispersed silver is 50 percent, what is the effective conductivity of this paste?

**2.20** **Ag–Ni alloys (contact materials) and the mixture rules** Silver alloys, particularly Ag alloys with the precious metals Pt, Pd, Ni, and Au, are extensively used as contact materials in various switches. Alloying Ag with other metals generally increases the hardness, wear resistance, and corrosion resistance at the expense of electrical and thermal conductivity. For example, Ag–Ni alloys are widely used as contact materials in switches in domestic appliances, control and selector switches, circuit breakers, and automotive switches up to several hundred amperes of current. Table 2.15 shows the resistivities of four Ag–Ni alloys used in make-and-break as well as disconnect contacts with current ratings up to ~100 A.