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Chapter 3

3.1 Photon energies in the visible and UV ranges

a. The human eye can typically see light in the wavelength range from around 400 nm (violet) to roughly 700 nm (red). What is the range of photon energies (in eV)?

b. The UV (ultraviolet) spectrum typically ranges from 100 nm to 400 nm. What is the photon energy range?

c. UVA, UVB and UVC correspond to wavelengths 100 – 280 nm, 280 – 315 nm and 315 – 400 nm respectively .What are the corresponding photon energy ranges?

Solution

Given: $\lambda = \lambda_{\text{violet}} = 400$ nm, the corresponding photon energy E_{ph} is,

$$E_{\text{ph}} = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\ \text{J s})(2.9979\times10^{8}\ \text{m s}^{-1})}{(400\times10^{-9}\ \text{m})} \times \frac{1}{1.602\times10^{-19}\ \text{J eV}^{-1}} = \mathbf{3.10\ eV}$$

We can calculate the photon energies corresponding to other wavelengths and build a table as in Table 3Q01-1.

Table 3Q01-1: Photon energies for different wavelength spectra

	Wavelengths	**E_{ph} (eV)**	**Definition of spectral range**
a.	λ = 400 – 700 nm	**3.10 – 1.77**	Visible range of the eye from violet to red
b.	λ = 100 – 400 nm	**12.4 – 3.10**	UV spectrum
c.	λ = 100 – 280 nm	**12.4 – 4.43**	UVA
c.	λ = 280 – 315 nm	**4.43 – 3.94**	UVB
c.	λ = 315 – 400 nm	**3.94 – 3.10**	UVC

3.2 Photons and photon flux

a. Consider a 1 kW AM radio transmitter at 700 kHz. Calculate the number of photons emitted fro-m the antenna per second.

b. The average intensity of sunlight on Earth's surface is about 1 kW m^{-2}. The maximum intensity is at a wavelength around 800 nm. Assuming that all the photons have an 800 nm wavelength, calculate the number of photons arriving on Earth's surface per unit time per unit area. What is the magnitude of the electric field in the sunlight?

Table 3.6

θ	0°	45°	90°	135°
λ' (nm)	0.0709	0.0715	0.0731	0.0749
$\Delta\lambda$ (nm)	0	0.0006	0.0022	0.004

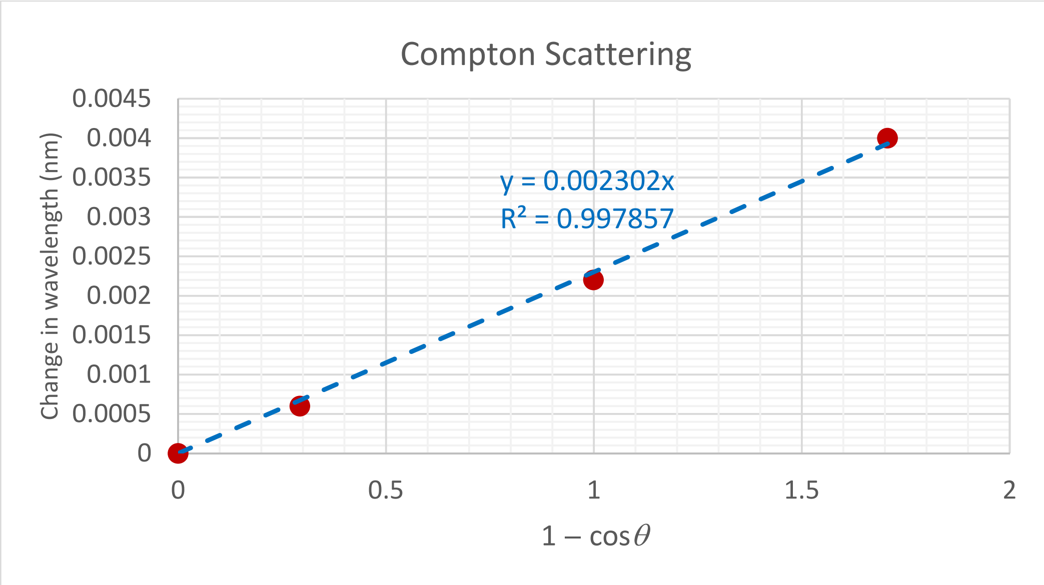


Figure 3Q09-1

Figure 3Q09-1 shows Δλ vs. 1 – cosθ with the best line forced through zero (why?). The slope is 0.00230 nm.

The slope from Figure 3Q09-1 is 0.00230×10^{-9} m, which, from Equation (6b), is given by

$$\text{Slope} = \lambda_C = \frac{h}{m_e c} = 0.00230\times10^{-9}$$

∴ $$h = m_e c \times \text{Slope} = (9.109\times10^{-31}\ \text{kg})(2.9979\times10^{8}\ \text{m s}^{-1})\times 0.00230\times10^{-9}\ \text{m}$$

∴ $$\boldsymbol{h = 6.281\times10^{-38}\ \text{J s (or kg m}^2\ \text{s}^{-1})}$$

Experimental discrepancy = 100×(6.626 – 6.281)/6.626 = 5.2%

Note: In addition to experimental errors, as we will see in Chapter 4, the electron inside a solid does not have its mass in vacuum – it has an effective mass. m_e above should be the electron effective mass.

3.10 Photoelectric effect A photoelectric experiment indicates that violet light of wavelength 420 nm is the longest wavelength radiation that can cause photoemission of electrons from a particular multialkali photocathode surface.

a. What is the work function of the photocathode surface, in eV?

b. If a UV radiation of wavelength 300 nm is incident upon the photocathode surface, what will be the maximum kinetic energy of the photoemitted electrons, in eV?

c. Given that the UV light of wavelength 300 nm has an intensity of 20 mW/cm^2, if the emitted electrons are collected by applying a positive bias to the opposite electrode, what will be the photoelectric current density in mA cm^{-2} ?

Solution

a. We are given $\lambda_{\max} = 420$ nm. The work function is then,

$$\Phi = hf_o = hc/\lambda_{\max} = (6.626 \times 10^{-34}\ \text{J s})(3.0 \times 10^8\ \text{m s}^{-1})/(420 \times 10^{-9}\ \text{m})$$

$\therefore$ $\mathbf{\Phi = 4.733 \times 10^{-19}}$ **J** or **2.96 eV**

b. Given $\lambda = 300$ nm, the photon energy is then:

$$E_{\text{ph}} = hf = hc/\lambda = (6.626 \times 10^{-34}\ \text{J s})(3.0 \times 10^8\ \text{m s}^{-1})/(300 \times 10^{-9}\ \text{m})$$

$\therefore$ $E_{\text{ph}} = 6.626 \times 10^{-19}$ J = **4.14 eV**

The kinetic energy *KE* of the emitted electron can then be found:

$$\boldsymbol{KE} = \Phi - E_{\text{ph}} = 4.14\ \text{eV} - 2.96\ \text{eV} = \mathbf{1.18\ eV}$$

c. The photon flux Γ_{ph} is the number of photons arriving per unit time per unit area. If I_{light} is the light intensity (light energy flowing through unit area per unit time) then,

$$\Gamma_{\text{ph}} = \frac{I_{\text{light}}}{E_{\text{ph}}}$$

Suppose that each photon creates a single electron, then

$$J = \text{Charge flowing per unit area per unit time} = \text{Charge} \times \text{Photon Flux}$$

$\therefore$ $$\boldsymbol{J} = e\Gamma_{\text{ph}} = \frac{eI_{\text{light}}}{E_{\text{ph}}} = \frac{(1.602\times10^{-19}\ \text{C})(200\ \text{W m}^{-2})}{(6.626\times10^{-19}\ \text{J})} = \mathbf{48.4\ A\ m^{-2} = 4.84\ mA\ cm^{-2}}$$

3.11 Photoelectric effect and quantum efficiency Cesium metal is to be used as the photocathode material in a photoemissive electron tube because electrons are relatively easily removed from a cesium surface. The work function of a clean cesium surface is 1.9 eV.

a. What is the longest wavelength of radiation which can result in photoemission?

b. If blue radiation of wavelength 450 nm is incident onto the Cs photocathode, what will be the kinetic energy of the photoemitted electrons in eV? What should be the voltage required on the opposite electrode to extinguish the external photocurrent?

c. **Quantum efficiency** (*QE*) of a photocathode is defined by,

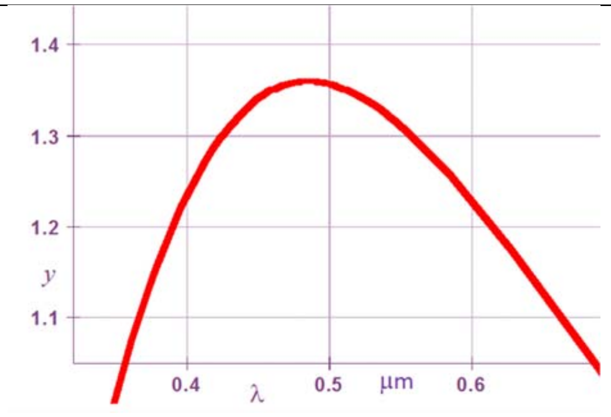


Figure 3Q13-5 Plot of $y = I_\lambda/I_T$ versus λ in microns. Maximum at around 480 nm.

RED, T = 2700 K

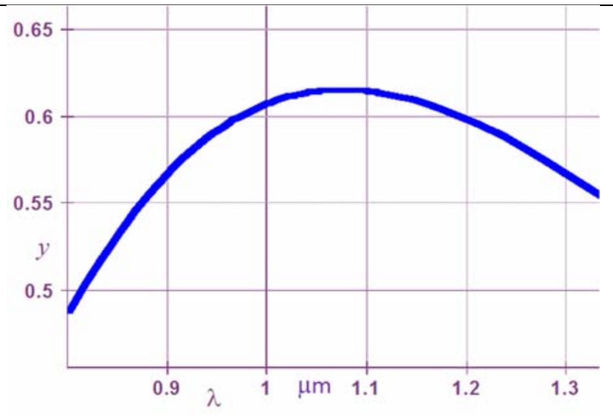


Figure 3Q13-6 Plot of $y = I_\lambda/I_T$ versus λ in microns. Maximum at around 1.07 μm.

BLUE, T = 6000K

2. We can link this problem to Question 3.15. The 40 W light bulb there has a tungsten filament, which is 0.381 m long and has a diameter of 33 μm. Surface area of the filament $S = 2\pi(D/2)L = 2\pi \times (33/2)\times10^{-6}$ m × 0.381 m = 3.950 × 10^{-5} m^2. Roughly, 40 W of electrical inout is radiated away. The emissivity of the tungsten surface is 0.35 which means it is a factor of 0.35 of the equivalent black body at the same temperature. Thus the black body would be emitting a total intensity of

$$I_T(\text{black body}) = \frac{P_{\text{bulb}}}{\varepsilon S} = \frac{40\ \text{W}}{(0.35)(3.95\times10^{-5}\ \text{m}^2)} = 2.9\times10^6\ \text{W m}^{-2}$$

which is very close to the I_t (= 3.0×10^6 W m^{-2}**)** we calculated at 2700 K. The answer to Question 3.15, by the way, is 2700 K.

3.14 Wien's law The maximum in the spectral intensity distribution of black body radiation depends on the temperature. Substitute $x = \lambda kT/hc$ in Planck's law in Equation 3.9 and plot it as a function of x and find $\lambda_{\max}$ which corresponds to the peak of the distribution, and hence derive Wien's law. Find the peak intensity wavelength $\lambda_{\max}$ for a 40 W light bulb given that its filament operates at roughly 2400 °C.

Solution

Let, $x = \lambda kT/hc$

$$I_\lambda = \frac{2\pi hc^2}{\lambda^5\left[\exp\left(\dfrac{hc}{\lambda kT}\right)-1\right]} = \frac{C}{x^5[\exp(1/x)-1]} \qquad (1)$$

where $C = 2\pi(kT)^5/h^4c^3$. (2)

Obviously C depends on kT but x in Equation 1 is essentially a variable that depends on λ. At T = 2400 °C, in SI units,

$$C = 8.3\times10^{10}\ \text{W m}^{-3} = 83\ \text{W mm}^{-3}$$

The units of C imply that it represents watts of radiation power emitted per m^2 of surface area and per m of wavelength interval; remember $I_\lambda \delta\lambda$ is the radiation intensity in the interval $\delta\lambda$. We have normalized the variable λ by dividing by hc/kT and assigning a variable x to this new normalized quantity. The constant C does not affect the shape of the function in x in the right hand side of Equation (1). Figure 3Q14-1 shows the plot of I_λ vs. x with C = 83 W mm^{-3}. The maximum is at $x = x_{max}$ = 0.20. We can easily demonstrate that x_{max} does not depend on the temperature. Figure 3Q14-2 shows the plots of I_λ vs. x with C = 83 W mm^{-3} and with C = 0.083 W mm^{-3} on log-log axes. It is clear that x_{max} = 0.2, does not depend on C and hence T.

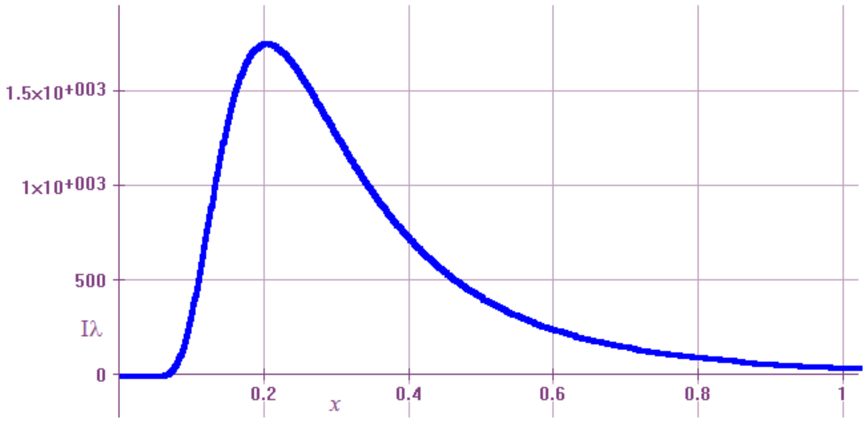


Figure 3Q14-1: Plot of I_λ versus x with C = 83 W mm^{-3}

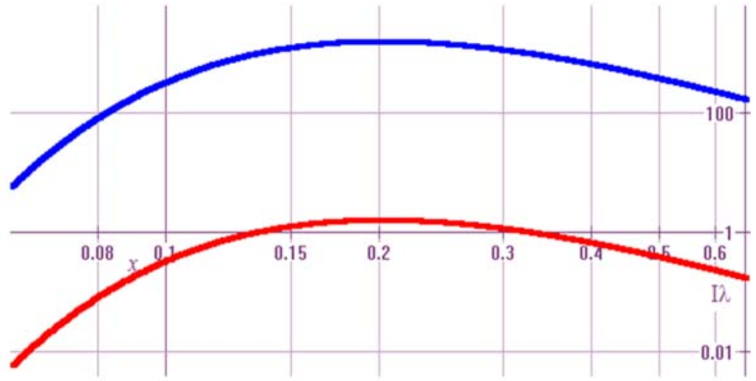


Figure 3Q14-2: Log-log plots of I_λ versus x for two values of C: Upper, C = 83 W mm^{-3} (BLUE), lower, C = 0.083 W mm^{-3} (RED)

At peak intensity, x_{max} = 0.20 (see Figures 3Q14-1 and 3Q14-2)

∴ $\lambda_{max}kT/hc \approx 0.20$

∴ $(1.38\times10^{-23}\text{ JK}^{-1})(\lambda_{max}T) / (6.626\times10^{-34}\text{ Js})(3\times10^{8}\text{ ms}^{-1}) = 0.20$

∴ $\boldsymbol{\lambda_{max}T = 2.88\times10^{-3}}$ **m K**

Given T = 2400 °C, we have

$\lambda_{max} = 2.88\times10^{-3}$ m K$/T = 2.88\times10^{-3}$ m K/2673 K = $\mathbf{1.07\times10^{-6}}$ **m = 1.07 μm**

This is the maximum in the spectral intensity I_λ vs. wavelength curve.

Note: The peak wavelength λ_{max} = 1.07 μm is not the same wavelength where the spectral intensity (intensity per unit frequency) in the frequency domain peaks; the latter is at a photon energy of 0.65 eV (See Question 3.13).

ADDENDUM

It is possible to get an analytical solution for the $\lambda_{max}T$ product by differentiating I_λ and setting it to zero. Consider differentiating I_λ, and setting it to zero.

Location of the peak intensity per unit wavelength at a given temperature.

Differentiate I_λ with respect to λ and set it to zero. Equivalently, differentiate y with respect to x below where x = λkT/hν

$$y = \frac{1}{x^5\left(\exp\left[\frac{1}{x}\right] - 1\right)}$$

$$\frac{d}{dx}y$$

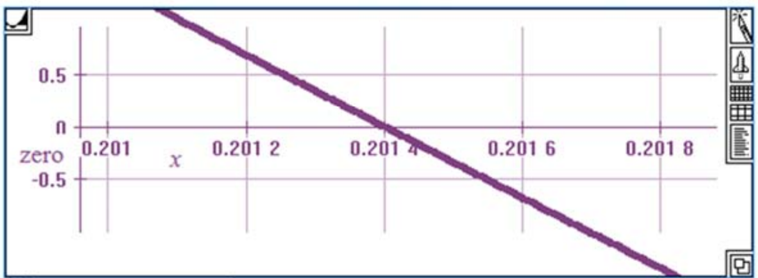
$$\frac{d}{dx}y = -\left(5x^4\left[e^{\frac{1}{x}} - 1\right] - e^{\frac{1}{x}}x^3\right)\frac{1}{\left(x^5\left[e^{\frac{1}{x}} - 1\right]\right)^2} \quad \textit{Substitute}$$

$$\frac{d}{dx}y = 5\frac{1}{x^6\left(-2e^{\frac{1}{x}} + e^{2\frac{1}{x}} + 1\right)} + \frac{e^{\frac{1}{x}}}{x^7\left(-2e^{\frac{1}{x}} + e^{2\frac{1}{x}} + 1\right)} - 5\frac{e^{\frac{1}{x}}}{x^6\left(-2e^{\frac{1}{x}} + e^{2\frac{1}{x}} + 1\right)} \quad \textit{Expand}$$

We can simplify this further and set it to zero to find the corresponding x at this point, that is, to find $x = x_{max}$ where $dI_\lambda / d\lambda = 0$.

Simplify the above and set it to zero, that is define zero as

$$\text{zero} = 5 + \frac{e^{\frac{1}{x}}}{x} - 5e^{\frac{1}{x}}$$



$x = 0.20141$

$\text{zero} = 0$

The slope gives the electron's wavelength. Next we calculate the expected De Broglie wavelength λ_{DB} of the electron (behaving as a wave) when the electron has been accelerated by a 10 kV anode voltage, that is KE = 10 keV. From Equation 3.16, we have

$$\lambda_{DB} = \frac{h}{p} = \frac{h}{(2em_eV)^{\frac{1}{2}}} = \frac{(6.626\times10^{-34}\text{ J s})}{[2(1.602\times10^{-19}\text{ C})(9.109\times10^{-31}\text{ kg})(10{,}000\text{ V})]^{\frac{1}{2}}}$$

∴ λ_{DB} = **0.0122 nm**

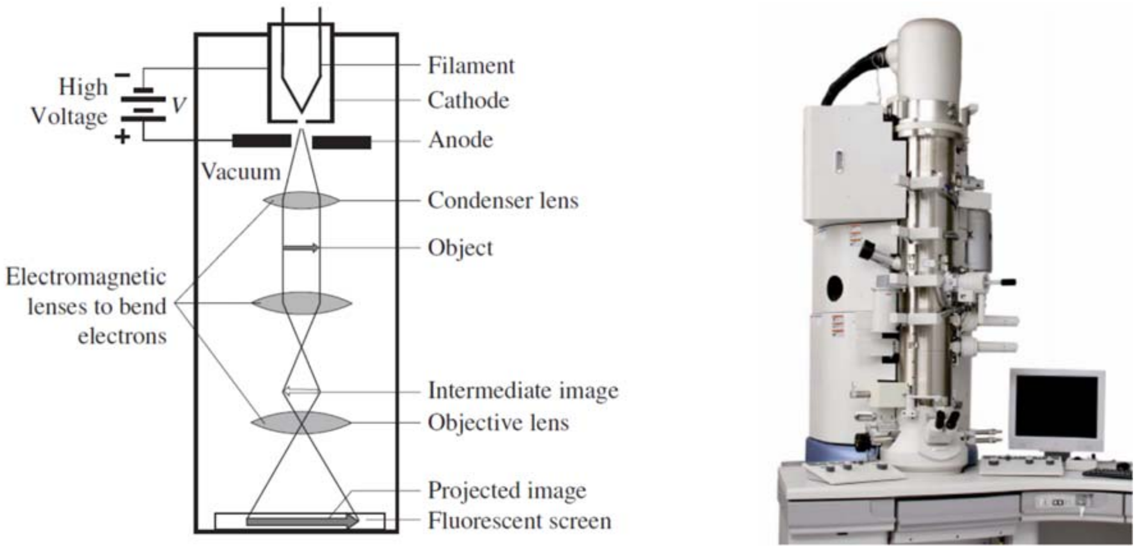
The discrepancy or the difference is 3.9%

***3.17 Electron microscope** Diffraction of light by an object becomes important when the wavelength of light is comparable to the object we wish to see. The resolution of an optical microscope cannot therefore be better than the wavelength of visible light, on the order of 500 nm. An electron microscope uses an electron beam (just like light) to "see" small objects because we can make the wavelength of an electron beam very short by adjusting the accelerating voltage. The transmission electron microscope (TEM) is an equipment that allows examining thin slices (or films) of materials under very large magnifications, for example 100,000× or more. As depicted in Figure 3.52, the image formation is exactly the same as that in the optical microscope except that electromagnetic coils acting as electron lenses are used to bend the electron ray. Electrons emitted by the hot cathode are accelerated by the anode which has typically a large voltage such as 100 kV applied to it with respect to the cathode. After passing through the anode, the electrons are collimated into a parallel beam by the condenser lens to be transmitted through the thin sample. An objective lens focuses the transmitted beam onto an intermediate image which is then projected on to a fluorescent screen by the projector lens. The whole apparatus operates under vacuum to avoid collisions of electrons with air molecules. The samples are typically less than 100 nm thick.

a. Do you need the wave properties of the electron to explain the operation of the electron microscope? (Explain your answer and consider whether you need interference and diffraction of waves to explain the optical microscope).

b. If the operating voltage of a transmission electron microscope is 100 kV, what is velocity of the electrons and their wavelength? (Neglect relativistic effects.)

c. Diffraction effects are negligible when the size of the object d is much greater than the wavelength λ of the wave. For example, the Bragg diffraction condition has no solutions when $2d > \lambda$. Resolution is therefore comparable in magnitude to the wavelength λ. What is the theoretical resolution, in order of magnitude, of the electron microscope operating at 100 kV and 300 kV? What do you think limits the resolution in practice?



(a) A schematic diagram of a transmission electron microscope. The angles of the electron trajectories with the optical axis are highly exaggerated; they are typically much less than 1°

(b) A Hitachi Transmission electron microscope (HF3300) with an accelerating voltage of 330 kV, maximum magnification of 1.5×10^6× and capable of resolving 0.13 nm. (Courtesy of Hitachi High-Technologies America Inc.

Figure 3.52 Transmission electron microscope

Solution

a. The operation of an optical microscope can be explained very simply by using **geoteric optics**, that is, representing light in terms of rays, which are then bent by lenses. Geometric optics works well when we can neglect the intereference and diffraction of light waves, that is when the light wavelength (λ) is much greater than the object features (say d) or $\lambda > d$. Similarly, we can explain the operation of an electron microscope treating the electron as **a particle moving along a trajectory**, i.e. along a line. The trajectory is bent by electrogmatic lenses just as rays are bent by lesnes. We do not need intereference or diffraction to explain the basic principle of operation and the origin of magnification.

We do however need the wave nature of the electron to explain the **resolution** of the electron microscope. Once the object feature becomes small, waves can be diffracted. Bragg's diffraction law $2d\sin\theta = \lambda$ (for first order) shows that diffraction cannot occur whenever $\sin\theta > 1$ or whenever $\lambda/2d > 1$ or $\lambda > 2d$. Put diffrently, this inequality allows us to estimate the resolution limit of a microscope, whether optical or electron. We cannot resolve objects of size smaller than the wavelength of light in the optical miscroscope or electron in the electron microscope.

The momentum of electrons can be evaluated from the accelerating voltage V because the kinetic energy gained by the electrons, ($p^2/2m_e$), is equal to eV. This in turns makes it possible to adjust the wavelength of electrons by adjusting the accelerating voltage

b. The voltage 100 kV (10^5 V) accelerates the electron to a KE equal to eV. From $KE = p^2 / 2m_e = eV$, we have

$$p = \sqrt{2m_e eV} = \sqrt{2 \times 9.109 \times 10^{-31} \times 1.602 \times 10^{-19} \times 10^5}$$

or $$p = \mathbf{1.709 \times 10^{-22}\ kg\ ms^{-1}}$$

The momentum $p = m_e v_{\text{electron}}$

∴ $v_{\text{electron}} = p / m_e$

or $v_{\text{electron}} = 1.7085 \times 10^{-22} / 9.109 \times 10^{-31}$ = **1.876 × 10⁸ m s⁻¹**

This velocity is actually is enormous because

$v_{\text{electron}} / c = 0.626$

Clearly, we cannot neglect relativistic effects as we have done in calculating v_{electron}.

The wavelength $\lambda = h / p$

$\lambda = 6.626 \times 10^{-34} / 1.709 \times 10^{-22}$ = **3.88×10^{-12} m or 3.88 pm**

c. The **100 kV** case was calculated above in *b* which gave λ = 3.88 pm, which is an estimate of the *theoretical* resolution of this electron microscope

The **300 kV** case leads to an electron momentum given by

$$p = \sqrt{2m_e eV} = \sqrt{2 \times 9.109 \times 10^{-31} \times 1.602 \times 10^{-19} \times 3 \times 10^5}$$

or p = **2.959×10^{-22} kg ms^{-1}**

The De Broglie wavelength $\lambda = h / p$

$\lambda = 6.626 \times 10^{-34} / 2.959 \times 10^{-22}$ = **2.24×10^{-12} m or 2.24 pm**

We know that the lens aberrations can limit the resolution of an optical microscope before the theoretical limit is reached ($d \sim \lambda/2$) . Similar problems arise in the electron microscope. It is not possible to design a perfect electromagnetic lens that can perfectly focus the electron trajectories. The bending ability of an electromagnet depends on the magnetic field and hence on the current, which has to be precisely controlled. Further, the focusing ability of an electromagnet will *also* depend on the velocity of the electrons. Not all the electrons have exactly identical velocities with a precise value. Small variations in electron velocities will mean different bending trajectories through the electromagnet lens; this is similar to chromatic dispersion in the optical microscope in which different wavelengths experience different amount of bending. Last, but not least, high resolution electron microscopes have be operated on a vibration-free (vibration-damped) floor. The electron microscope above in the photo operates at 330 kV but has a quoted resolution of 0.13 nm; compare with the calculation above.

3.18 Heisenberg's uncertainty principle. Show that if the uncertainty in the position of a particle is on the order of its de Broglie wavelength, then the uncertainty in its momentum is about the same as the momentum value itself.

Solution

The de Broglie wavelength is

$$\lambda = \frac{h}{p}$$

where p is the momentum. From Heisenberg's uncertainty principle we have,

$$\Delta x \Delta p \approx \hbar$$

If we take the uncertainty in the position to be of the order of the wavelength, $\Delta x \sim \lambda$, then

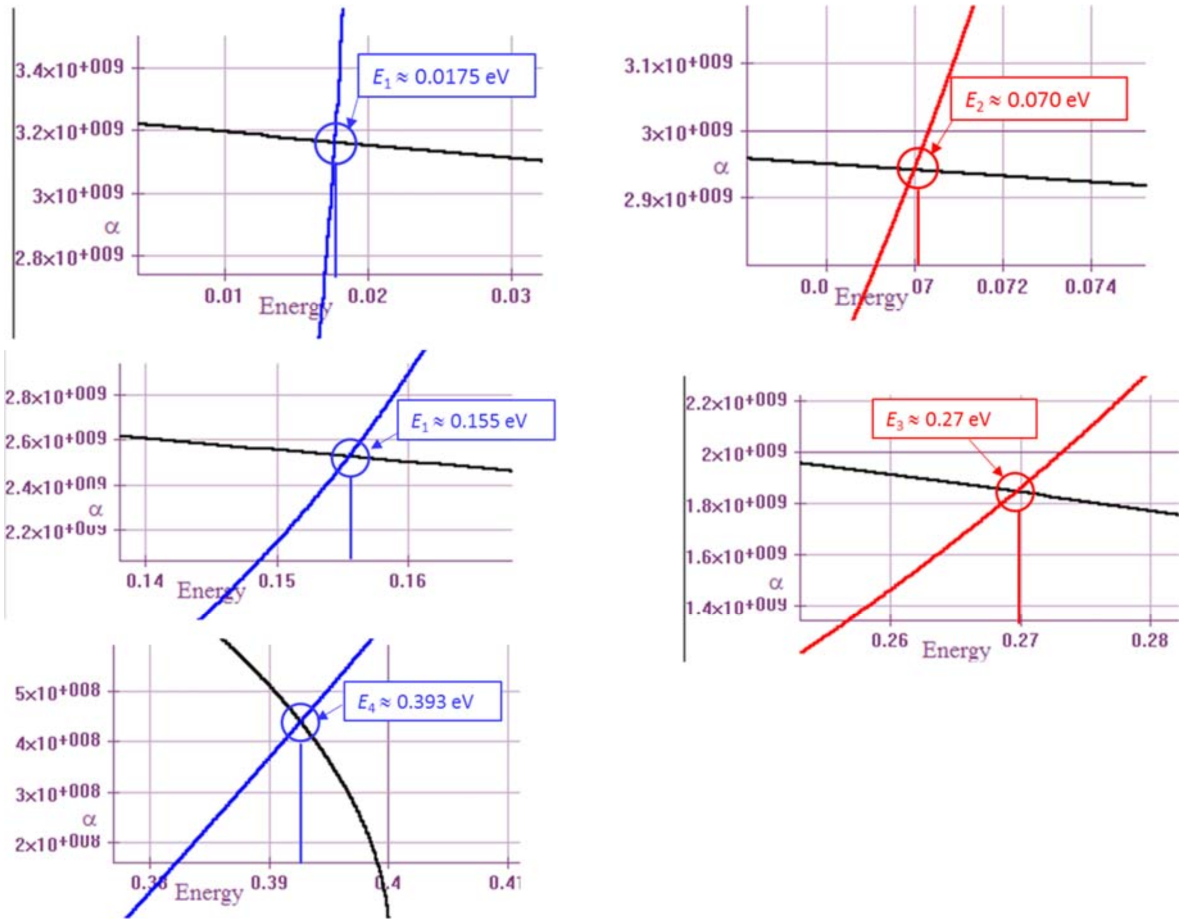


Figure 3Q21-3: Solution of $\alpha = k\tan(ka/2)$ (blue) and $\alpha = -k\cot(ka/2)$ (red) with intersection points in Figure 3Q21-2 expanded.

Note: Solving

$$\alpha = k\tan(\tfrac{1}{2}ka)$$

means that we are looking for a solution, a value of E, that satisfies

$$f(E) = \alpha - k\tan(\tfrac{1}{2}ka) = 0$$

in which

$$\alpha^2 = \frac{2m_e(V_o - E)}{\eta^2} \qquad \text{and} \qquad k^2 = \frac{2m_eE}{\eta^2}$$

We can start by guessing a solution by using a value form an infinite PE well, $E = E_{1\infty}$, and then, by trial and error (*e.g.* using a Secant or Newton-Raphson Method), finding the solution E_1. With powerful math software these days, graphical solutions as in Figure 3Q21-3 can be obtained quickly and allow the visualization of all possible solutions.

3.22 Tunneling

a. Consider the phenomenon of tunneling through a potential energy barrier of height V_o and width a, as shown in Figure 3.19. What is the probability that the electron will be reflected? Given the transmission coefficient T, can you find the reflection coefficient R? What happens to R as a or V_o or both become very large?

b. For a wide barrier ($\alpha a >> 1$), show that T_o can at most be 4 and that $T_o = 4$ when $E = \frac{1}{2} V_o$.

Solution

a. The relative reflection probability or reflection coefficient R is given as the ratio of the square of the amplitude of the reflected wave to that of the incident wave, which is:

$$R = \frac{A_2^2}{A_1^2}$$

Also, R can be found from the transmission coefficient T by the equation $R = 1 - T$, as stated in Equation 3.47. From Equation 3.43, T is given as:

$$T = \frac{1}{1 + D(\sinh[a\alpha])^2}$$

where a is the width of the potential energy barrier, α is the rate of decay, and D is given by:

$$D = \frac{V_o^2}{4E(V_o - E)}$$

To determine the behavior of R as a or V_o or both become very large, we can use the equation $R = 1-T$ to express R in terms of a and D (remember D is a function of V_o).

$$R = 1 - T = 1 - \frac{1}{1 + D(\sinh[a\alpha])^2}$$

$\therefore$
$$R = \frac{D(\sinh[a\alpha])^2}{1 + D(\sinh[a\alpha])^2}$$

We know that $\sinh(\infty) = \infty$, and also that $1 / \infty = 0$. Therefore, as V_o becomes large, so does D, which leads to $T = 0$ and $R = 1$, meaning total reflection occurs. If a becomes large then $\sinh(\infty) = \infty$ and $T = 0$, making $R = 1$ for total reflection.

b. We need to find the maximum value of T_o. Since T_o depends on the energy E, we can differentiate it with respect to E, set the result to 0 and isolate E.

$$T_o = \frac{16E(V_o - E)}{V_o^2} \qquad \text{(See Equation 3.46)}$$

$\therefore$
$$\frac{dT_o}{dE} = 16\left(\frac{-2E + V_o}{V_o^2}\right) = 0$$

$\therefore$
$$E = \frac{1}{2}V_o$$

Thus T_o is maximum when $E = V_o / 2$. If this expression for energy is substituted back into the equation for T_o to find its maximum value (T_o'):

$$T_o' = \frac{16E(V_o - E)}{V_o^2} = \frac{16\left(\frac{1}{2}V_o\right)\left(V_o - \frac{1}{2}V_o\right)}{V_o^2} = \frac{16V_o^2}{4V_o^2} = \mathbf{4}$$

***3.23 Three dimensional quantum well** Consider the energy of an electron in a 3D cubic *PE* well in which the electron energy is given by Equation 3.52. If we measure the energy ε normalized to the E_{111} level , then

$$\varepsilon = \frac{E}{E_{111}} = n_1^{\,2} + n_2^{\,2} + n_3^{\,2} = N^2$$

corresponding to the wavefunction in Equation 3.51 with $a = b = c$.

a. Consider the case $n_1 = 5$, $n_2 = 2$, $n_3 = 1$ or $N^2 = 30$. How many wavefunctions are there? What is the degeneracy of this energy level?

b. Suppose that we wish to find the total number, that is the sum S, of all wavefunctions with energies less than some critical energy ε'. We need all n_1, n_2, n_3 combinations that would give $\varepsilon = n_1^{\,2} + n_2^{\,2} + n_3^{\,2} < \varepsilon'$. Consider "*n*-space" in which n_1, n_2, n_3 are variables corresponding to *x, y, z*, and we take n_1 along x, n_2 along y, and n_3 along z. $N'^2 = n_1'^2 + n_2'^2 + n_3'^2 = \varepsilon'$ represents those n_1, n_2, n_3 values that give ε'. What is $x^2 + y^2 + z^2 = \varepsilon'$ in this *n*-space space? What does the volume of space in this sphere located so that *x*, *y* and *z* are all positive represent? This volume is 1/8th of the volume of the sphere with radius ε', that is, $S = (1/8)(4\pi/3)\varepsilon'^{3/2}$. What does this represent? If we differentiate this with respect to energy, $dS/d\varepsilon'$, what would we get? Can we use it to represent a density of states in energy?

Solution

Part I

The value $N^2 = 30$ can be obtained from (5,2,1) as well as (5,1,2), (2,1,5), (2,5,1), (1,2,5) and (1,5,2). There are thus six states from the combination of (5,2,1) giving six possible states, each with a distinct wave function , ψ_{n1n2n3}. However, all these, ψ_{n1n2n3} have the same energy E_{512}

Part II

$N'^2 = n_1'^2 + n_2'^2 + n_3'^2 = \varepsilon'$ represents those n_1, n_2, n_3 values that give ε'. What is $x^2 + y^2 + z^2 = \varepsilon'$ in this *n*-space space? This is a **sphere** in which

$$n_1'^2 + n_2'^2 + n_3'^2 \le \varepsilon' \qquad (1)$$

What does the volume of space in this sphere located so that *x*, *y* and *z* are all positive represent? What does the volume of space in this sphere located so that *x*, *y* and *z* are all positive represent? T**his volume represents all possible combinations of n_1,n_2,n_3 that satisfy Equation 1**. This volume is 1/8th of the volume of the sphere with radius ε', that is,